Exact Theory of Interdigital Band-Pass Filters and Related Coupled Structures

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Abstract—An exact theory of interdigital line networks and related coupled structures is presented. The theory of parallel-coupled line arrays is reviewed briefly, and the derivation of exact equivalent circuits from the impedance matrix using modern network synthesis techniques is discussed.

A simplified theory of equivalent coupled structures is introduced in order to avoid the lengthy analysis required when using the impedance matrix approach. Equivalent networks for the interdigital line are obtained by inspection, using a transformed capacitance matrix associated with the two-dimensional geometry of the conductors and ground planes. The techniques presented are simple to apply and allow a given transmission response to be obtained in a variety of line configurations. A practical design example and experimental results are given to illustrate the simplicity of the approach, along with general criteria for the design of practical filter networks with optimum transmission characteristics.

The paper is directed toward the design of interdigital band-pass filters; however, the techniques presented can be used to analyze and design a much broader class of microwave networks. The relationship of the exact theory to existing approximate theory is discussed.

Glossary

b = ground plane spacing

β = phase constant for TEM propagation along a length of transmission line

$\beta l=\theta$ = electrical length of a transmission line in radians

c' = total static capacitance between conductors per unit length along the conductors

$c=c'/\epsilon$ = ratio of static capacitance between conductors per unit length to the permittivity of the dielectric medium (this ratio is independent of the dielectric medium and depends only on cross-sectional geometry)

C = characteristic admittance of an open-circuited quarter-wavelength line (an S-plane capacitor)

ϵ_r = relative dielectric constant

f = frequency

f_0 = frequency at which a transmission line is a quarter wavelength

I = current

k = an integer used to denote position in a series

$K_{k,k+1}$ = voltage coupling factor between the kth and $(k+1)$st line in an N-line array of parallel conductors

L = characteristic impedance of a short-circuited quarter-wavelength line (an S-plane inductor)

L-C = network elements corresponding to S-plane capacitors C and inductors L

m = number of nonredundant L-C type elements in an optimum filter

n = number of nonredundant unit elements in an optimum filter prototype

N = number of conductors in a parallel array (an integer)

n' = factor used to transform capacitance matrix

η_0 = characteristic impedance of free space (376.7 ohms/square)

S-plane = transformed complex frequency plane in which distributed elements are treated as lumped elements

$S=j\tan\ \beta l=j\tan\ \pi f/2f_0$ = transformed frequency variable

s/b = normalized coupled bar line spacing

t/b = normalized coupled bar line thickness

u.e. = unit element, a quarter wavelength of transmission line of characteristic impedance Z_0

V = voltage

w/b = normalized coupled bar line width

z_{11}, z_{22} = open-circuit driving point impedance of a two-port network

z_{12} = open-circuit transfer impedance of a two-port network

Z_0 = characteristic impedance in ohms

Z_{0k} = characteristic impedance of the kth line in the presence of the other open-circuited lines in an N-line parallel array of infinite length conductors.

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I. Introduction

APPROXIMATE design techniques for interdigital filters have been presented by Matthaei [1] and have been used to design both narrow-band and wide-band filters. While these techniques are sufficiently accurate for many engineering applications, an exact theory is desirable when optimum network performance or special network configurations are required. The use of exact synthesis procedures allows greater flexibility in design and physical form, and makes possible a more accurate determination of practical circuit limitations.

The work described in this paper is presented in the following manner: In Section II the basic theory of interdigital structures is reviewed briefly. In Section III a general synthesis procedure starting with the impedance matrix of a parallel-coupled line array is described. This procedure can be used to derive an exact cascade equivalent circuit suitable for obtaining optimum network designs. However, because of the tedious nature of this method, equivalent circuits for the interdigital filter are obtained by using a computationally simpler approach, described in Section IV. A simplified theory of equivalent coupled structures based on the two-dimensional geometry of the conductors and ground planes is presented (Section IV). By using the techniques described, exact equivalent circuits of interdigital structures are readily derived. The relatively simple methods can be used to obtain physical realizations of an optimum filter prototype in the form of coaxial structures, interdigital structures, or combinations of the specified network forms.

In Section V a practical design example and experimental results are given which illustrate the simplicity and flexibility of the equivalent network design approach. In Section VI general design criteria for practical network configurations are discussed, along with the relationship of the exact theory to Matthaei's approximate design procedure.

II. Parallel Line Arrays

The basic network to be considered in obtaining an exact equivalent circuit for the interdigital line is the $2N$ port consisting of an array of N-parallel open-circuited conductors between ground planes, as presented in Fig. 1. Bolljahn and Matthaei [2] have shown how to obtain the impedance matrix for an array of parallel conductors all of which have the same characteristic impedance Z_0. The general matrix for an array of conductors with different characteristic impedances Z_{0k} can be obtained by a simple extension of their method. The general network equations in matrix form can be shown to be

$$\begin{bmatrix} V_{1A} \\ V_{2A} \\ \vdots \\ V_{NA} \\ V_{1B} \\ V_{2B} \\ \vdots \\ V_{NB} \end{bmatrix} = \frac{1}{S} \left[\begin{array}{cccc|cccc} Z_{01} & Z_{01}K_{12} & \cdots & Z_{01}K_{1N} & Z_{01}\sqrt{1-S^2} & Z_{01}K_{12}\sqrt{1-S^2} & \cdots & Z_{01}K_{1N}\sqrt{1-S^2} \\ Z_{02}K_{21} & Z_{02} & \cdots & Z_{02}K_{2N} & Z_{02}K_{21}\sqrt{1-S^2} & Z_{02}\sqrt{1-S^2} & \cdots & Z_{02}K_{2N}\sqrt{1-S^2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ Z_{0N}K_{N1} & Z_{0N}K_{N2} & \cdots & Z_{0N} & Z_{0N}K_{N1}\sqrt{1-S^2} & Z_{0N}K_{N2}\sqrt{1-S^2} & \cdots & Z_{0N}\sqrt{1-S^2} \\ \hline Z_{01}\sqrt{1-S^2} & Z_{01}K_{12}\sqrt{1-S^2} & \cdots & Z_{0N}K_{1N}\sqrt{1-S^2} & Z_{01} & Z_{01}K_{12} & \cdots & Z_{01}K_{1N} \\ Z_{02}K_{21}\sqrt{1-S^2} & Z_{02}\sqrt{1-S^2} & \cdots & Z_{02}K_{2N}\sqrt{1-S^2} & Z_{02}K_{21} & Z_{02} & \cdots & Z_{02}K_{2N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ Z_{0N}K_{N1}\sqrt{1-S^2} & Z_{0N}K_{N2}\sqrt{1-S^2} & \cdots & Z_{0N}\sqrt{1-S^2} & Z_{0N}K_{N1} & Z_{0N}K_{N2} & \cdots & Z_{0N} \end{array}\right] \begin{bmatrix} I_{1A} \\ I_{2A} \\ \vdots \\ I_{NA} \\ I_{1B} \\ I_{2B} \\ \vdots \\ I_{NB} \end{bmatrix} \quad (1)$$

The characteristic impedance Z_0 of a lossless uniform transmission line operating in the TEM mode is related to its shunt capacitance [3] by

$$Z_0 = \frac{\eta_0}{\sqrt{\epsilon_r}(c'/\epsilon)} = \frac{\eta_0}{\sqrt{\epsilon_r}(c)}. \quad (2)$$

The dimensionless ratio $c = c'/\epsilon$ simplifies the symbols required and can be used directly with Getsinger's [3] or Cristal's [4] charts to obtain physical dimensions.

The characteristic impedances Z_{0k} and coupling factors $K_{k,k+1}$ are determined in terms of the dimensionless self and mutual static capacitances per unit length, c, which exist between the open-circuited lines and ground, as shown in Fig. 2(a). The Z_{0k} values are obtained by substituting in (2) the total capacitance c_{0k} from the kth line to ground. The total capacitance can be obtained by series-parallel reduction. The coupling factors between any two lines, k and $k+1$, can

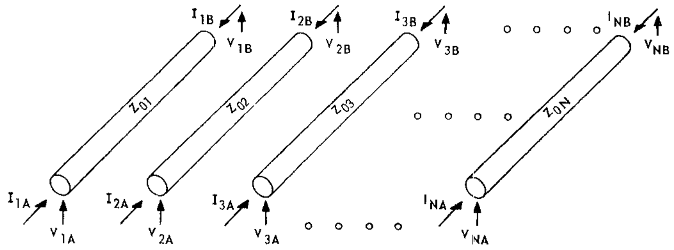


Fig. 1. Basic parallel-coupled line array showing important network variables.

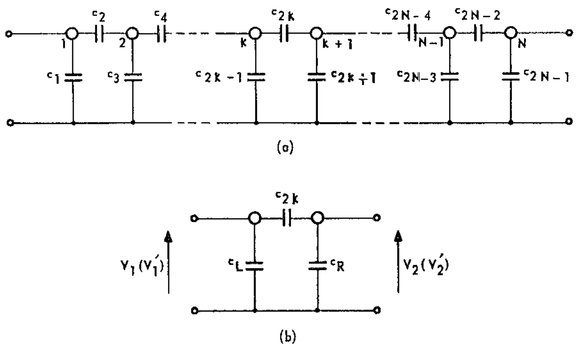


Fig. 2. Two-dimensional capacitance array of the interdigital line.

be computed by replacing the capacitance to the left of line k by an equivalent capacitor c_L, and the capacitance to the right of line $k+1$ by a capacitor c_R, as shown in Fig. 2(b). The values of c_L and c_R are obtained, as before, by series-parallel reduction. The coupling factors are then given by

$$K_{k,k+1} = \frac{V_2}{V_1} = \frac{\frac{1}{c_R}}{\frac{1}{c_R}+\frac{1}{c_{2k}}} = \frac{c_{2k}}{c_{2k}+c_R},$$

$$K_{k+1,k} = \frac{V_1'}{V_2'} = \frac{\frac{1}{c_L}}{\frac{1}{c_L}+\frac{1}{c_{2k}}} = \frac{c_{2k}}{c_{2k}+c_L}, \qquad (3)$$

where V_1 is the applied voltage and V_2 is the coupled voltage when determining $K_{k,k+1}$, and V_2' is the applied voltage and V_1' is the coupled voltage when determining $K_{k+1,k}$. Note that in general $K_{k,k+1} \neq K_{k+1,k}$.

The representation of the coupled array by the circuit of Fig. 2(a) assumes that no significant direct coupling exists between nonadjacent lines. Coupling of this type would require the addition of capacitors between nonadjacent lines and would greatly complicate the analysis and synthesis problem. As pointed out by Bolljahn [2] and Matthaei [1], [2], for many practical line configurations, the neglect of coupling of this type is not a serious limitation. Assuming no direct coupling between nonadjacent conductors, the higher-order coupling factors that appear in (1) are obtained by multiplying the lower-order coupling factors. For example,

$$K_{k,k+3} = (K_{k,k+1})(K_{k+1,k+2})(K_{k+2,k+3}). \qquad (4)$$

Finally, reciprocity requires that terms located in a manner symmetric about the principal diagonal of the impedance matrix of (1) be equal, i.e., that $z_{ik} = z_{ki}$ for $i \neq k$. Therefore,

$$Z_{01}K_{12} = Z_{02}K_{21}, \qquad Z_{01}K_{13} = Z_{03}K_{31},$$
$$Z_{02}K_{23} = Z_{03}K_{32}, \text{ etc.} \qquad (5)$$

Further details concerning the network properties of parallel-coupled line arrays can be found in References [1] and [2].

III. Equivalent Circuit of the Interdigital Line Using Modern Network Synthesis Techniques

The array of conductors in Fig. 1 becomes an interdigital line by alternately shorting and leaving open opposite ends of each conductor. The array can begin with either an open-circuited or a short-circuited line [1]. The input and output ports are on the same side of the array for N odd and on opposite sides for N even. A general network approach that can be used to obtain an exact equivalent circuit for the interdigital filter is as follows:

1) The pertinent port conditions, $V=0$ at a short circuit and $I=0$ at an open circuit, are applied to matrix equation (1) to obtain the network equations for the corresponding interdigital structures.
2) The resulting equations are solved simultaneously and reduced to a 2×2 impedance matrix so as to recognize the two-port impedance parameters z_{11}, z_{12}, and z_{22}.
3) Modern network synthesis techniques are used to obtain a network that realizes the specified impedance parameters.

For the general case, the $[z]$ parameters obtained by carrying out steps 1) and 2) cannot be realized in a network form that involves only L's, C's, and ideal transformers (where L and C refer to the impedance and admittance of a short-circuited and open-circuited quarter wavelength of transmission line, respectively [5]). As will be shown, the interdigital filter with open-circuited terminating lines has a third-order pole at dc corresponding to three L-C elements, and the interdigital line with short-circuited terminating lines has a first-order pole at dc corresponding to one shunt L. The remaining N-3 elements for the N-line network with open-circuited terminating lines and N-1 elements for the network with short-circuited terminating lines are unit elements (u.e.).

The analysis and synthesis procedure described in steps 1) through 3) has been carried out for symmetric networks with up to eight lines, and for a general asymmetric network with up to four lines.[1] This process is long and tedious, and becomes intractable for networks with a large number of lines. As network equiv-

[1] A detailed discussion of the synthesis procedure, including examples, is given in Reference [6].

alences were obtained, a definite pattern was noticed in the resulting equivalent relationships; this pattern led to an alternate method of derivation. The main value of the impedance matrix approach is that it provides an alternate derivation method—and thus a check on the technique to be presented in Section IV.

IV. Simplified Theory of Equivalent Coupled Structures

The electrical behavior of an array of parallel conductors can be described either by matrix equation (1) or by the capacitance network of Fig. 2(a). In this section, the capacitance network and its admittance-derived matrix will be used to obtain equivalent circuits in a manner computationally simpler than that of Section III. The techniques to be presented will also provide insight into the physical form of equivalent networks.

A. General Network Relations

Before discussing specific applications, some general relations involving capacitance networks in the form of Fig. 2(a) will be presented.[2] A five-node network will be used to illustrate the method. The extension to more complex networks follows in a straightforward manner. Referring to Fig. 2(a), with $N=5$, the capacitance matrix[3] is given by

$$\begin{bmatrix} c_1+c_2 & -c_2 & 0 & 0 & 0 \\ -c_2 & c_2+c_3+c_4 & -c_4 & 0 & 0 \\ 0 & -c_4 & c_4+c_5+c_6 & -c_6 & 0 \\ 0 & 0 & -c_6 & c_6+c_7+c_8 & -c_8 \\ 0 & 0 & 0 & -c_8 & c_8+c_9 \end{bmatrix}. \quad (6)$$

Multiplying the kth row and column of this matrix by any factor n' corresponds to multiplying the admittance level of the kth node by n'^2, and the transfer admittance between nodes adjacent to the kth node by n'. If this transformation is performed on any interior row and column, the two-port performance of the network is unchanged; if performed on an end row and column, the admittance level of the corresponding port is multiplied by n'^2 and the overall transfer admittance is multiplied by n'. Performing multiple transformations allows an infinite number of equivalent networks and port admittance levels to be obtained. In performing these transformations, care must be taken to insure that the final circuit contains no negative elements. Physical realizability requires that the sum of the elements of any row or column be greater than (or equal to) zero, and gives a limitation on the choice of n'. For example, multiplying the second row and column of matrix (6) by n' leads to the constraints

$$n'^2(c_2+c_3+c_4) \geq n'(c_2+c_4)$$
$$c_1+c_2 \geq n'c_2. \quad (7)$$

These require that n' be confined to the range

$$\frac{c_2+c_4}{c_2+c_3+c_4} \leq n' \leq \frac{c_1+c_2}{c_2}. \quad (8)$$

An important aspect of transformations on the capacitance matrix is that the product of the coupling factors, having indices which are interchanged, remains invariant,[4] i.e.,

$$(K_{k,k+1})(K_{k+1,k}) = K^2(\text{invariant}). \quad (9)$$

Multiplying a row and column by n' corresponds to introducing an admittance level change of n'^2 and results in changing $K_{k,k+1}$ to $n'(K_{k,k+1})$ and $K_{k+1,k}$ to $(1/n')(K_{k+1,k})$. Consequently, once any pair of coupling factors is found, those for all equivalent networks may readily be determined.

B. Forms of Coupling

In carrying out these transformations, the capacitors are treated as though they represent static capacitances in a planar geometry. From a microwave viewpoint, they represent coupling between quarter-wave lines, and if the form of the coupling could be determined, the microwave equivalent circuit could be obtained by inspection. Consider, for example, the single-line network of Fig. 3(a), page 565, and its capacitor representation in Fig. 3(b). The static capacitor of Fig. 3(b) can represent three distinct forms of coupling, depending on the terminal conditions and the terminals between which the electrical performance is observed:

1) An S-plane capacitor[5]—the electrical behavior of the line at either port with the other port open-circuited. The value of the S-plane capacitor is given by $C=1/Z_0=\sqrt{\epsilon_r}c/\eta_0$, where Z_0 is the characteristic impedance of the line as defined by (2).
2) An S-plane inductor—the electrical behavior of the line at either port with the other port short-circuited. The value of the inductor is given by $L=Z_0=\eta_0/\sqrt{\epsilon_r}c$.
3) A unit element—the electrical behavior of the line between the ports. The impedance of the unit element is equal to $Z_0=\eta_0/\sqrt{\epsilon_r}c$.

[2] The basic approach taken is based on the discussion of equivalent networks given in Reference [7], pp. 141–176.

[3] The diagonal terms in the capacitance matrix are given by the sum of the capacitors directly connected to a node, while the off-diagonal terms are the capacitors directly connected between nodes. The matrix is related to the short-circuit admittance matrix, resulting in negative off-diagonal terms [7]

[4] A proof of the invariance of the product of the coupling factors follows in a straightforward manner by applying the transformations to a generalized array and computing the new coupling factors. A discussion of the importance of this property is given in the Appendix.

[5] A detailed discussion of S-plane elements and their properties is given in Reference [5].

If the general array of Fig. 2(a) can be put into a form in which the previously mentioned types of coupling become easily recognizable, an equivalent S-plane network can be found by inspection. Transformations of the capacitance matrix preserve the performance of the network, with the possible exception of a change in impedance level, and offer the possibility of changing the network form into one in which the nature of each coupling is easily recognizable. Furthermore, the capacitance network also represents static capacitors and can be used to obtain dimensions for a physical realization.

To observe the effect that transformations of the capacitance matrix have on the physical form of the network, consider the symmetric capacitor circuit shown in Fig. 4(a) and the interdigital line shown in Fig. 4(b). For simplicity, all the capacitor values are chosen to be unity. The capacitance network indicates that there is coupling from all lines to ground and between adjacent lines as is evident from the physical configuration. The capacitance matrix is

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{bmatrix}. \tag{10}$$

Multiplying the second row and column of the matrix by n' leaves the two-port properties invariant, but yields an equivalent network with a capacitance matrix given by

$$\begin{bmatrix} 2 & -n' & 0 \\ -n' & 3n'^2 & -n' \\ 0 & -n' & 2 \end{bmatrix}. \tag{11}$$

Physical realizability requires that

$$3n'^2 \geq 2n' \quad \text{and} \quad 2 \geq n'$$

or

$$\frac{3}{2} \leq n' \leq 2. \tag{12}$$

Suppose an n' of 5/3 is chosen. The resulting capacitance matrix is given by

$$\begin{bmatrix} 2 & -\frac{5}{3} & 0 \\ -\frac{5}{3} & \frac{25}{3} & -\frac{5}{3} \\ 0 & -\frac{5}{3} & 2 \end{bmatrix}. \tag{13}$$

The corresponding capacitance network, shown in Fig. 4(c), indicates that the coupling between the ground planes and the outer conductors has decreased, whereas the coupling between conductors, and the coupling between the center conductor and ground, has increased. A physical network that would accomplish this change in coupling is shown in Fig. 4(d).

If a limiting value of $n'=2$ is chosen in the matrix (11), the resulting matrix becomes

$$\begin{bmatrix} 2 & -2 & 0 \\ -2 & 12 & -2 \\ 0 & -2 & 2 \end{bmatrix}. \tag{14}$$

The capacitance network and a physical realization are shown in Figs. 4(e) and 4(f), respectively. The outer lines are no longer coupled to the ground plane but only to the center line. The center line is coupled to both outer lines and to the ground plane. From Fig. 4(f), the form of each coupling is readily determined and an exact equivalent circuit can be written by inspection. The coupling between the outer lines and the center line (terminals ① – ② and ②–③) is that of a series S-plane capacitor. The coupling between the center line and ground (terminals ②–④) is that of an S-plane inductor. The equivalent circuit between the input and output ports is thus that of a three-element ladder, as shown in Fig. 4(g). Since the transformations performed do not change the two-port performance, the circuit of Fig. 4(g) is a general equivalent circuit[6] for networks with any choice of n'. The capacitor values in the planar representation can be linked directly to the L and C values of the S-plane equivalent circuit by (2) and the discussion at the beginning of Section IV-B. The transformation of matrices (10) through (14) can then be worked in reverse order to obtain an interdigital realization of the circuit rather than a coaxial one. The transformed capacitor values can be used directly [3], [4] to obtain line dimensions.

C. Interdigital Line Equivalent Circuits

The equivalent circuits for more complex interdigital lines are obtained in an analogous manner. To facilitate their derivation, it is useful to know a few of the general results concerning capacitance networks of the type under consideration.

A capacitance network containing both self and mutual capacitors can be obtained by transformation from a capacitance network having an odd number of nodes containing all series c's and one shunt c from the center line to the ground plane. Similarly, a capacitance network containing both self and mutual capacitors can be obtained by transforming a capacitance network having an even number of nodes containing all series c's and shunt c's from the two nodes nearest the center.[7] These statements are illustrated in Figs. 5(a) and 5(b) for the odd and even cases, respectively, and can be demonstrated by applying the transformations already outlined. The reason for using a network form containing a small number of shunt c's is that the nature of the coupling between lines is readily found in terms

[6] The circuit of Fig. 4(g) describes the transmission response for all line configurations but does not indicate the impedance level of the interior node.

[7] The transmission properties (for fixed load impedances) of all networks obtained in this manner are identical if the admittance levels of the outer nodes are not changed.

of S-plane elements. A cascade equivalent circuit that describes the transmission characteristics of all equivalent networks then can be obtained by inspection. There is no absolute reason for using the forms of Fig. 5 rather than, for example, those that have all series c's and one shunt c from an arbitrary node. However, the forms shown in Fig. 5 are particularly useful for symmetric networks and can often be used for asymmetric networks without the addition of ideal transformers.

An example derivation of the S-plane equivalent circuit will be given for each of the basic interdigital line configurations. For the first example, consider deriving the equivalent circuit of a five-line network with open-circuited terminating lines. The capacitance network form with all series c's and one shunt c is shown in Fig. 6(a), and a coaxial realization[8] is shown in Fig. 6(b). The lines and port conditions have been added to the capacitance array to aid in reasoning to the equivalent circuit. Referring to Fig. 6, the following three steps are taken to determine the equivalent circuit:

1′) Line ① couples only to line ②, and the coupling has the form of a series S-plane capacitor C_1 because terminal 1(b) is open. The distributed capacitor exists between terminals 1(a) and 2(a) and corresponds to a static capacitance c_2, from (2), given by $c_2=(\eta_0 C_1)/\sqrt{\epsilon_r}$. An identical situation holds between lines ④ and ⑤ and results in $c_8=(\eta_0 C_2)/\sqrt{\epsilon_r}$.

2′) Line ③ couples to ground, and the coupling has the form of an S-plane inductor of value L because of the shorted terminal 3(a). The inductor exists between terminals 3(b) and the ground plane, and corresponds to a static capacitance c_5, given by $c_5=\eta_0/\sqrt{\epsilon_r}L$.

3′) The coupling between lines ② and ③ (or ③ and ④) is in the form of a coaxial unit element with a 180° phase reversal. More specifically, consider lines ② and ③:

a′) End 2(a) is an ungrounded terminal at the (a) end of the u.e.

b′) End 3(a) is end 2(a)'s ground terminal.

c′) End 3(b) is the ungrounded terminal at the (b) end of the u.e.

d′) End 2(b) is its respective ground.

A diagram and equivalent circuit of the coupling between lines ② and ③ is shown in Fig. 6(c). Note the presence of the −1:1 ideal transformer to represent the 180° phase reversal. The static capacitors c_4 and c_6 (corresponding, respectively, to the unit-element characteristic impedance values Z_1 and Z_2) are given by

$$c_4 = \frac{\eta_0}{\sqrt{\epsilon_r} Z_1} \quad \text{and} \quad c_6 = \frac{\eta_0}{\sqrt{\epsilon_r} Z_2}\cdot$$

The complete equivalent S-plane circuit is shown in Figure. 6(d). The two −1:1 ideal transformers can be combined without changing the transmission characteristics and result in the final equivalence of Fig. 6(e).

[8] It is interesting to note the relation of the coaxial interdigital network realization to Cohn's [8] re-entrant section. The interdigital filter is seen to be similar to a multiple re-entrant section with suitable terminal conditions.

The coupling factors and characteristic impedances of the lines can be obtained quite simply from the array of Fig. 6(e) by using the appropriate c's in (2). For example,

$$Z_{01} = \frac{\eta_0}{\sqrt{\epsilon_r}\,(c_{01})} = \frac{\eta_0}{\sqrt{\epsilon_r}}\left(\frac{1}{c_2}+\frac{1}{c_4}+\frac{1}{c_5}\right)$$

$$= \frac{1}{C_1} + Z_1 + L$$

and similarly,

$$Z_{06} = Z_2 + L + \frac{1}{C_2}\cdot$$

For the coupling factors,

$$K_1^2 = (K_{12})(K_{21}) = \left\{\frac{\dfrac{1}{c_4}+\dfrac{1}{c_5}}{\dfrac{1}{c_2}+\dfrac{1}{c_4}+\dfrac{1}{c_5}}\right\}(1)$$

$$= \frac{Z_1 + L}{\dfrac{1}{C_1} + Z_1 + L} \tag{15}$$

and similarly,

$$K_2^2 = K_{23}K_{32} = \frac{L}{Z_1 + L}$$

$$K_3^2 = K_{34}K_{43} = \frac{L}{Z_2 + L}$$

$$K_4^2 = K_{45}K_{54} = \frac{Z_2 + L}{\dfrac{1}{C_2} + Z_2 + L}\cdot$$

The results obtained using the capacitance matrix approach are identical to those that were obtained using the method described in Section III. For the second example, consider the six-line network with short-circuited terminating lines. The capacitance network and a physical realization are shown in Figs. 7(a) and 7(b), respectively. The S-plane equivalent circuit with numbered terminal pairs is shown in Fig. 7(c), and the final equivalence is shown in Fig. 7(d). The steps followed in obtaining the equivalent circuit are similar to those of the first example, as are the methods for obtaining the Z_{0k} and coupling coefficients.

The equivalent circuits for interdigital line networks with any number of lines are given in Tables I and II, along with the relationships between the S-plane equivalent circuit and the capacitance network element values. It is interesting to note that the equivalent circuit for the N-2 line network with short-circuited terminating lines can be obtained by letting the input capacitors of the N-line network with open-circuited terminating lines become infinite. An infinite number of equivalent networks can be obtained by transforming the capacitance network, the final result of any set of transformations giving a set of self and mutual capacitors that can be used with Getsinger's [3] or Cristal's [4] charts to obtain physical dimensions. It is obvious that no preferred physical configuration exists. A practical realization may take forms ranging from an all-coaxial structure to an all-interdigital line, or combinations of the two. The physical form that is most practical depends on the network element values required for a given bandwidth and filter type.

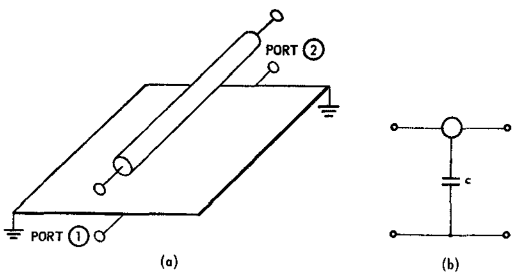


Fig. 3. Determination of S-plane elements for a single line above a ground plane.

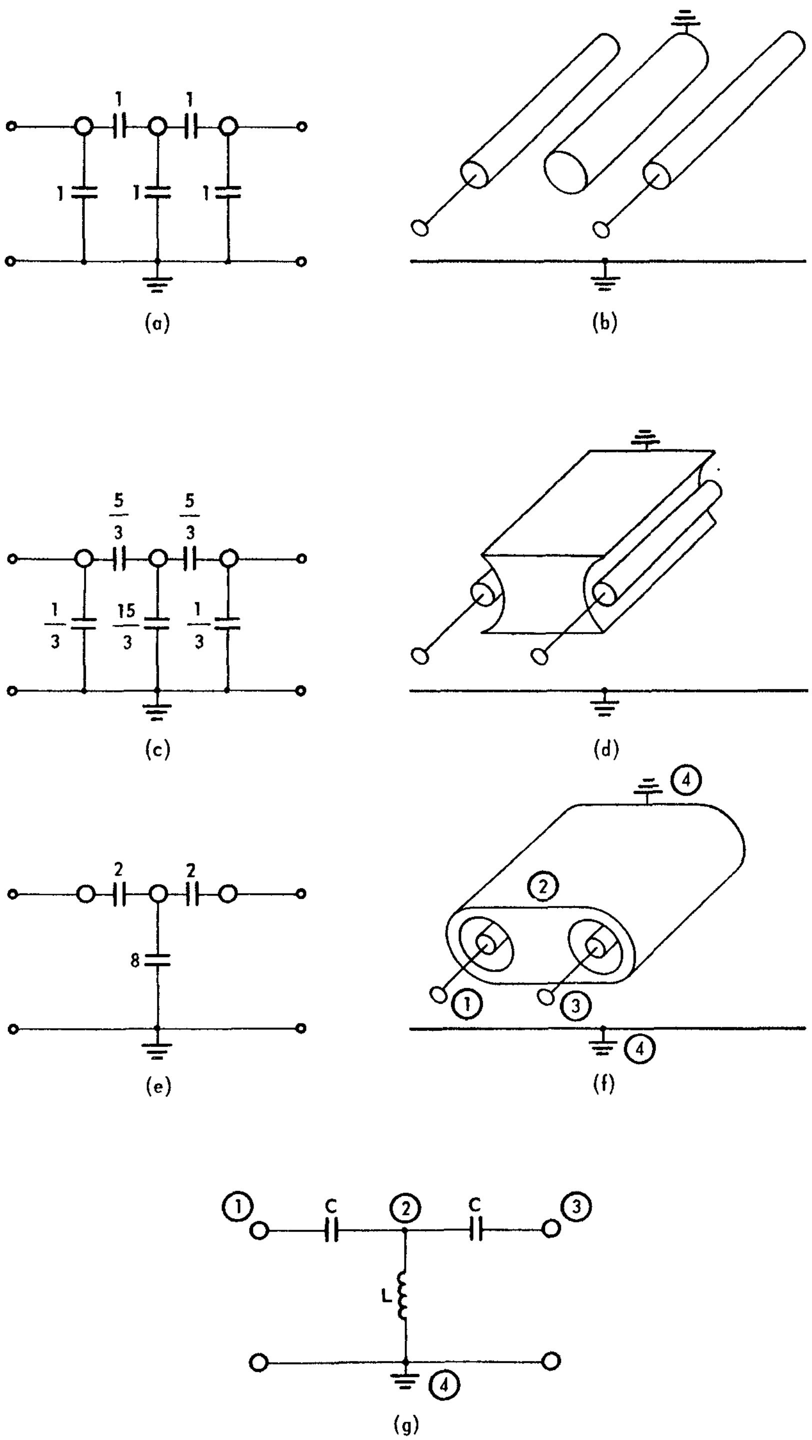


Fig. 4. Effect of transformations of the capacitance matrix on physical form.

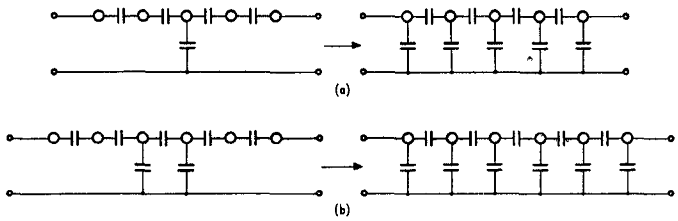


Fig. 5. Capacitance network equivalences.

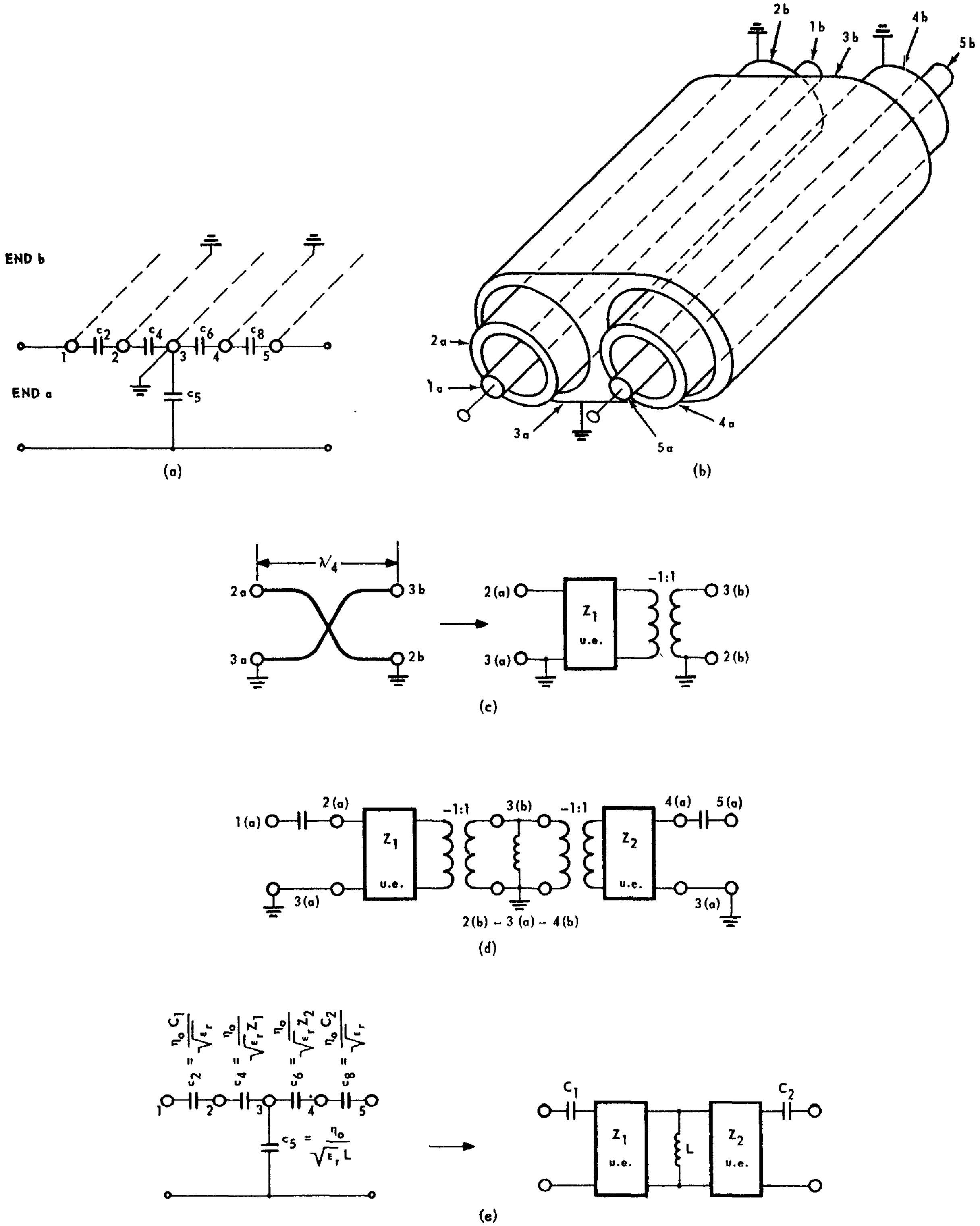


Fig. 6. Derivation of the S-plane equivalent circuit of the five-line interdigital network with open-circuited terminating lines.

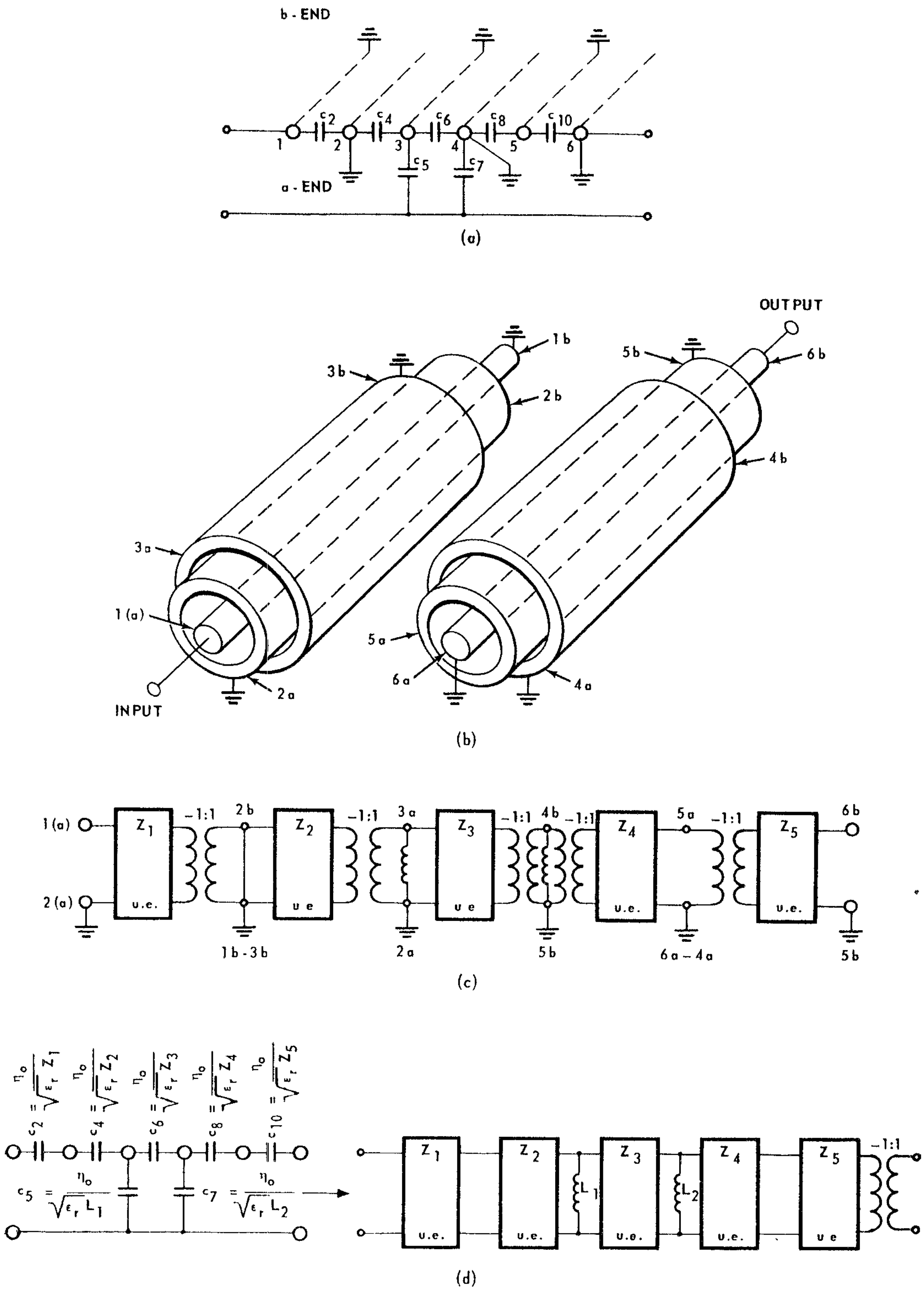
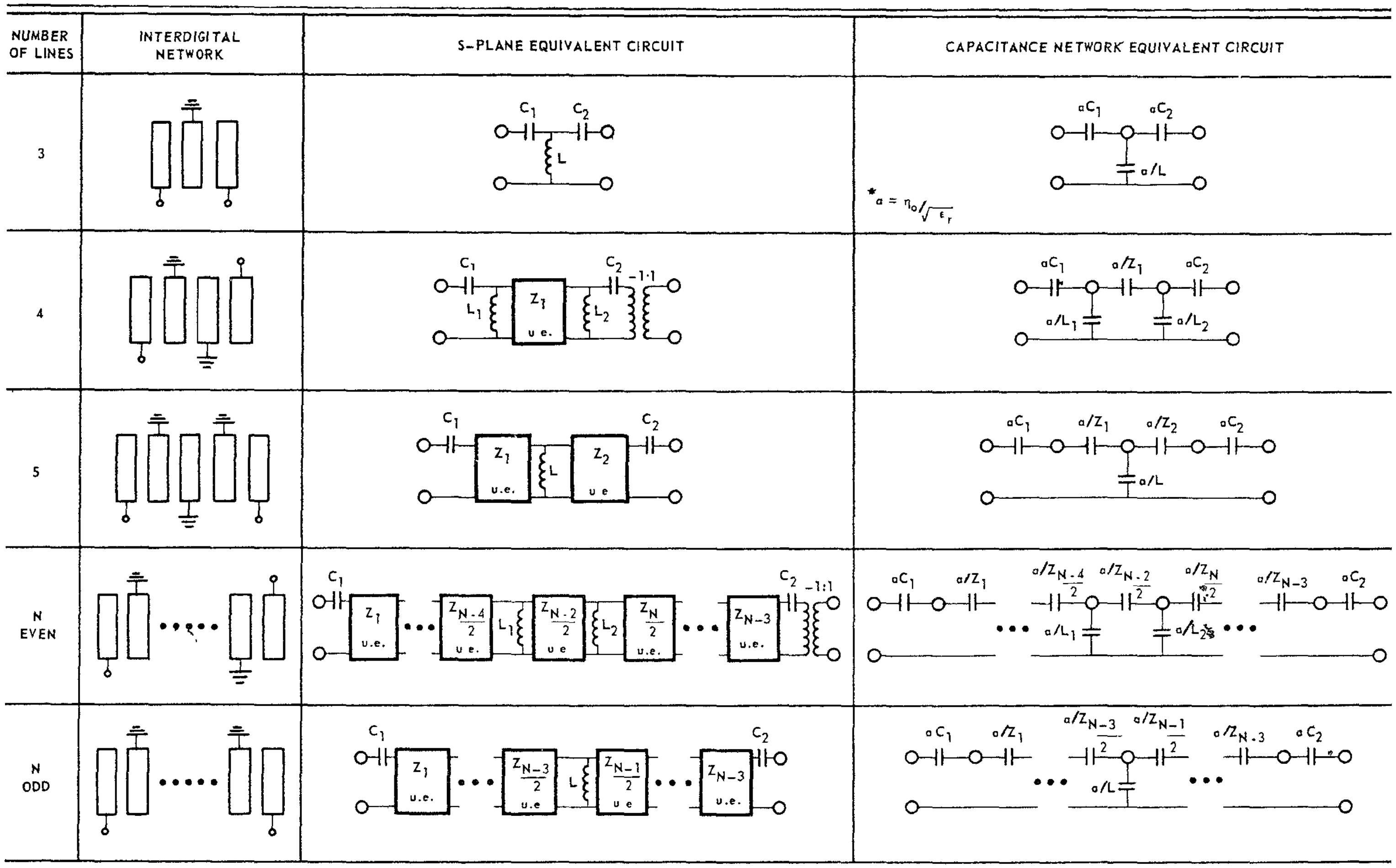


Fig. 7. Derivation of the S-plane equivalent circuit of the six-line interdigital network with short-circuited terminating lines.

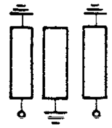
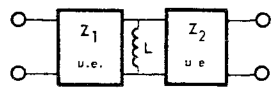
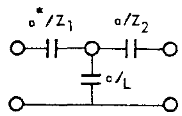
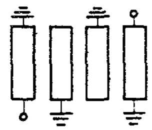
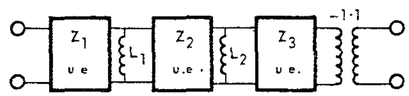
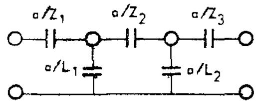
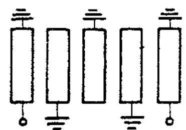
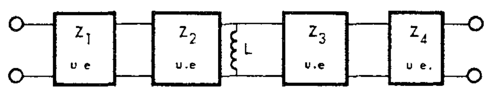
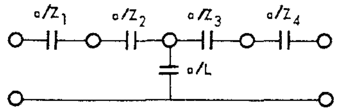
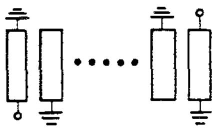
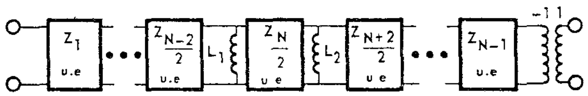
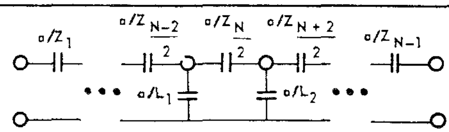
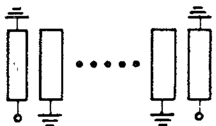
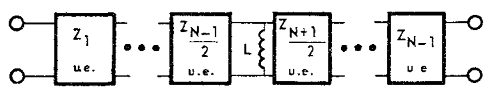
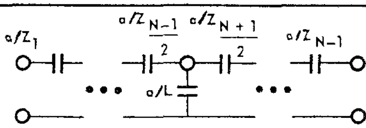
TABLE I

EQUIVALENT CIRCUITS FOR INTERDIGITAL NETWORKS WITH OPEN-CIRCUITED TERMINATING LINES



$^{*}a = \eta_0/\sqrt{\epsilon_r} = 7.54/\sqrt{\epsilon_r}$ FOR USE IN OBTAINING 50 OHM DESIGNS USING S-PLANE ELEMENT VALUES NORMALIZED TO 1 OHM

TABLE II
EQUIVALENT CIRCUITS FOR INTERDIGITAL NETWORKS WITH SHORT-CIRCUITED TERMINATING LINES

NUMBER OF LINES	INTERDIGITAL NETWORK	S-PLANE EQUIVALENT CIRCUIT	CAPACITANCE NETWORK EQUIVALENT CIRCUIT
3			 *a = η0/√εr
4			
5			
N EVEN			
N ODD			

$^*a = \eta_0/\sqrt{\epsilon_r} = 7.54/\sqrt{\epsilon_r}$ FOR USE IN OBTAINING 50 OHM DESIGNS USING S-PLANE ELEMENTS NORMALIZED TO 1 OHM

V. Design Example and Experimental Results

The S-plane element values for prototype networks such as those of Tables I and II must be determined in order to obtain a practical filter realizable in an interdigital or related form. The problem of determining element values that give an optimum response (in a Butterworth or Chebyshev sense) has been solved in detail by Horton and Wenzel [9], and the synthesis procedures described can be used to obtain tables of element values for a wide range of bandwidths and filter types.[9] Inspection of Tables I and II shows that the N-line filter with open-circuited terminating lines is an $m=3$, $n=N$-3 prototype, and the N-line filter with short-circuited terminating lines is an $m=1$, $n=N$-1 prototype. The number of nonredundant L-C elements and unit elements is given by m and n, respectively.[10]

To illustrate the simplicity and flexibility of the physical form incorporated in the design procedure, a six-section 3:1 bandwidth 0.05-dB-ripple Chebyshev filter will be described. The element values, shown (with the symmetric prototype) in Fig. 8(a), were obtained by synthesis using the procedure given in Reference [9]. The self and mutual static capacitor values are obtained using the relationships in Table I, and the corresponding capacitance network is shown in Fig. 8(b). The network could be constructed (depending on the practicality of resulting dimensions) in the coaxial form shown in Fig. 7(b) directly from the prototype element values. An interdigital realization is obtained by transforming the capacitance matrix corresponding to the array of Fig. 8(b). This matrix, obtained by inspection, is

$$\frac{7.54}{\sqrt{\epsilon_r}}\begin{bmatrix} 1.00 & -1.00 & 0 & 0 & 0 & 0 \\ -1.00 & 2.80 & -1.80 & 0 & 0 & 0 \\ 0 & -1.80 & 9.32 & -3.60 & 0 & 0 \\ 0 & 0 & -3.60 & 9.32 & -1.80 & 0 \\ 0 & 0 & 0 & -1.80 & 2.80 & -1.00 \\ 0 & 0 & 0 & 0 & -1.00 & 1.00 \end{bmatrix}. \tag{16}$$

$$\frac{7.54}{\sqrt{\epsilon_r}}\begin{bmatrix} 1 & -n'_1 & 0 & 0 & 0 & 0 \\ -n'_1 & 2.80n'^2_1 & -1.80n'_1n'_2 & 0 & 0 & 0 \\ 0 & -1.80n'_1n'_2 & 9.32n'^2_2 & -3.60n'^2_2 & 0 & 0 \\ 0 & 0 & -3.60n'^2_2 & 9.32n'^2_2 & -1.80n'_1n'_2 & 0 \\ 0 & 0 & 0 & -1.80n'_1n'_2 & -2.80n'^2_1 & -n_1 \\ 0 & 0 & 0 & 0 & -n'_1 & 1 \end{bmatrix} \begin{matrix} \\ \leftarrow \times n'_1 \\ \leftarrow \times n'_2 \\ \leftarrow \times n'_2 \\ \leftarrow \times n'_1 \\ \\ \end{matrix} \tag{17}$$

$$\begin{matrix} & \uparrow & \uparrow & \uparrow & \uparrow & \\ & \times n'_1 & \times n'_2 & \times n'_2 & \times n'_1 & \end{matrix}$$

The steps required to design an optimum filter in an interdigital or related form can be summarized as follows:

1) From tables[9] or by synthesis [9], obtain the S-plane element values (L, C, and Z).
2) Using the relationships in Tables I and II, obtain static capacitors for use directly with Getsinger's [3] or Cristal's [4] charts.
3) Equivalent networks that realize the same transmission response as the prototype network are obtained by applying the transformation presented in Section IV.

An all-interdigital realization can be obtained by multiplying the interior rows and columns by constants, as shown in matrix (17). The values of these constants are restricted by realizability conditions as discussed in Section IV, but are otherwise arbitrary. In general, the factors (n_i') that multiply the interior nodes need not be chosen in a symmetric manner; this is necessary only if a symmetric physical structure is desired. Different load admittance levels can be accommodated by changing the admittance levels of either or both outer nodes.

For the 3:1 bandwidth filter under consideration, the convenient value of the series capacitors in the prototype circuit allows them to be incorporated coaxially in the terminating lines of a four-line interdigital realization of the interior prototype elements. By incorporating the capacitors coaxially in the end elements, as shown in Fig. 8(c), relaxed mechanical tolerances are obtained as compared with those required for an all-interdigital realization, and the filter is more compact. With the capacitance matrix approach, the designer has

[9] Tables of element values for symmetric filters with equal terminating impedances are presently being computed and will be included in the Final Report of USAERDL Contract DA-28-043-AMC-00399(E). Element values for single terminated networks required in multiplexer [10] design can be obtained by letting the functions described in Reference [9] represent transfer impedances or admittances.

[10] A detailed discussion of optimum filter theory and the parameters m and n is given in Reference [9].

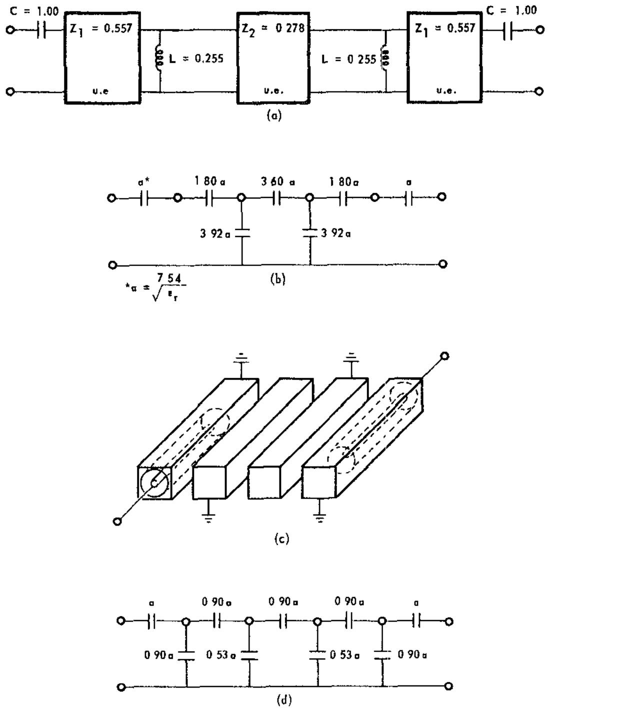


Fig. 8. Six-section 3:1 bandwidth 0.05-dB-ripple filter design. (a) Synthesized prototype. (b) Prototype capacitance matrix. (c) Coaxial-interdigital configuration. (d) Transformed capacitance matrix.

the freedom of using an essentially all-coaxial configuration, a combination coaxial-interdigital structure, or an all-interdigital structure as dicated by the practicality of the resulting physical dimensions.

Incorporating the series capacitors in the end elements of the interdigital interior section is equivalent to choosing $n'_1 = 1$. The interdigital sections can be designed by referring to the 4×4 interior submatrix of matrix (17) and choosing a suitable value of n'_2. One possibility is to choose n'_2 so that the line spacings for a rectangular bar realization are equal. This requires [3] the interior mutual capacitors to be equal, or from matrix (17)

$$
\begin{aligned}
3.60 n_2'^2 &= 1.80 n'_2 \\
n'_2 &= \tfrac{1}{2}.
\end{aligned}
\tag{18}
$$

The final capacitance matrix is given in (19), and the capacitance network is shown in Fig. 8(d). The dotted submatrices of matrices (17) and (19) show the interior 4×4 matrix pertinent to the interdigital sections:

For a rectangular bar realization, the normalized line spacings s/b are determined [1] directly from Getsinger's [3] chart and the normalized line widths w/b by the equations

$$
\begin{aligned}
\frac{w_1}{b} &= \frac{1}{2}\left(1 - \frac{t}{b}\right)\left[\frac{1}{2}(c_1) - c_{f'} - (c_{fe'})_{12}\right] \\
\frac{w_k}{b} &= \frac{1}{2}\left(1 - \frac{t}{b}\right)\left[\frac{1}{2}(c_k) - (c_{fe'})_{k-1,k} - (c_{fe'})_{k,k+1}\right] \\
\frac{w_N}{b} &= \frac{1}{2}\left(1 - \frac{t}{b}\right)\left[\frac{1}{2}(c_N) - c_{f'} - (c_{fe'})_{N-1,N}\right]
\end{aligned}
\tag{20}
$$

where c_k is the capacitance from the kth line to ground, and $c_{k,k+1}$ is the capacitance between the kth and $(k+1)$st lines. (Note the use of small c's consistent with the Glossary definitions.) A thorough discussion of the method of obtaining dimensions from a set of self and mutual capacitors, along with a detailed definition of the symbols in (20), is given in References [1], [3], and [11]. The dimensions of the coaxial end sections are readily determined from convenient graphs [12] and correspond on a normalized 1-ohm basis to

$$
Z_0 = \frac{\eta_0}{\sqrt{\epsilon_r}\, c_{/2}} = \frac{7.54}{\sqrt{\epsilon_r}\left(\frac{7.54}{\sqrt{\epsilon_r}}\right)} = 1
$$

A summary of the data used in designing the 3:1 bandwidth filter, along with calculated dimensions, is given in Table III.

An experimental model was constructed based on the data in Table III. The interdigital line sections were arbitrarily shortened by 0.200 inch, and tuning screws were added to obtain the proper resonator length. The desired length of the coaxial end elements was determined by temporarily shortening the interior lines with shorting blocks. This moved the singularity produced by the interior sections at twice the center frequency to a higher frequency, and allowed the series capacitors to be adjusted to proper length. After performing these operations the filter was tested. The bandwidth agreed with the theoretical to within 1 percent, but a reflection peak of 1.8 VSWR appeared near both upper and lower band edges. The addition of a 0.020-inch shim under the gap between the center lines and the adjustment of the tuning screws resulted in the final response, shown in Fig. 9. A detailed drawing of the filter is given in Fig. 10, and a photograph appears in Fig. 11. As can

$$
\frac{7.54}{\sqrt{\epsilon_r}}
\begin{bmatrix}
1 & -1 & 0 & 0 & 0 & 0 \\
-1 & 2.80 & -.90 & 0 & 0 & 0 \\
0 & -.90 & 2.33 & -.90 & 0 & 0 \\
0 & 0 & -.90 & 2.33 & -.90 & 0 \\
0 & 0 & 0 & -.90 & 2.80 & -1 \\
0 & 0 & 0 & 0 & -1 & 1
\end{bmatrix}
\tag{19}
$$

TABLE III

Design Data for a 3:1 Bandwidth Interdigital Filter

k^*	c_k	w_k (INCHES)	$c_{k,k+1}$	$s_{k,k+1}$ (INCHES)	NOTES
1	0	–	$\frac{7.54}{\sqrt{\epsilon_r}}$	–	c_{12} WAS REALIZED AS A TEFLON LOADED SERIES COAXIAL STUB, $\sqrt{\epsilon_r} = 1.44$, SEE FIG. 10
2	6.79	0.248	6.79	0.058	
3	4.02	0.195	6.79	0.058	THE DIMENSIONS ARE SYMMETRIC ABOUT THE CENTER OF THE FILTER

b = 0.625 INCHES
t = 0.375 INCHES

*THE k's REFER TO THE LINE NUMBERS IN FIG. 10. c_k IS THE CAPACITANCE FROM THE k^{th} LINE TO GROUND AND $c_{k,k+1}$ IS THE CAPACITANCE BETWEEN THE k^{th} AND $k+1^{st}$ LINES

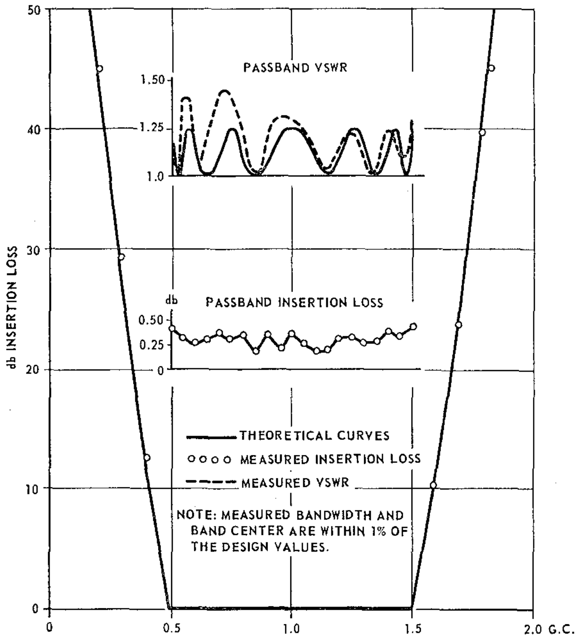


Fig. 9. Theoretical and measured performance of a six-section 3:1 bandwidth interdigital filter.

be seen from Fig. 9, the experimental response agrees very closely with the theoretical. The bandwidth and center frequency are within 1 per cent of the theoretical, and the maximum VSWR was 1.45:1 as compared with a theoretical maximum of 1.25:1. The close agreement between measured and theoretical VSWR curves and experimental bandwidth is a good check on the accuracy of the design theory.

VI. Criteria for Practical Network Design

The techniques presented in Section V can be used to design a wide variety of filter types and to obtain many different physical realizations. No one physical form is best suited to all bandwidths and all filter types. It may be necessary to investigate a number of equivalent forms to obtain practical dimensions. Fortunately, by using the techniques presented, this can be accomplished in a relatively simple manner. The following general criteria[11] are given to aid in obtaining a practical design.

[11] The criteria presented are directed toward symmetric filters, but similar results hold true for asymmetric networks.

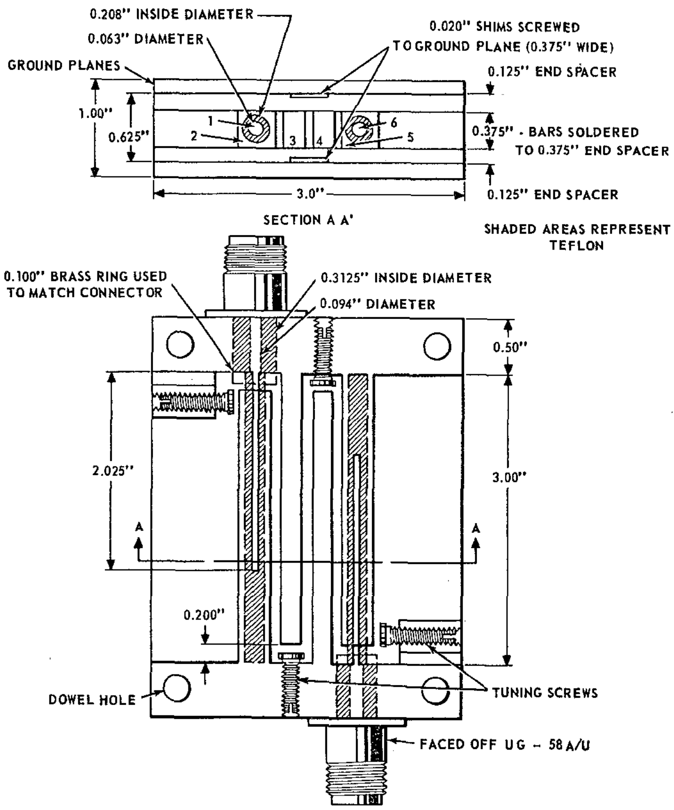


Fig. 10. Drawing of an experimental 3:1 bandwidth interdigital filter. (Other dimensions are given in Table III.)

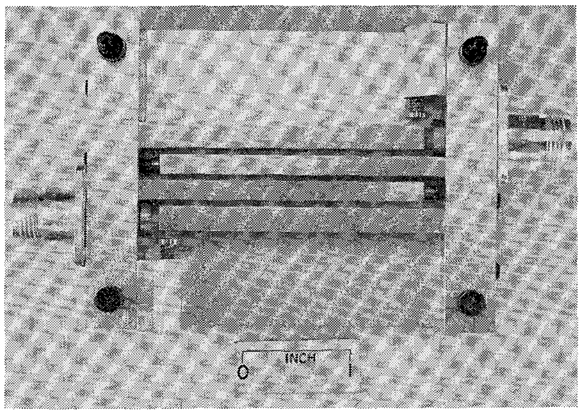


Fig. 11. A 3:1 bandwidth interdigital filter.

1) *Filters of moderate to wide bandwidths:* As pointed out in Reference [9], the filter properties of L-C type elements are superior to unit elements, the improvement becoming greater as filter bandwidth increases. The interdigital filter form with open-circuited terminating lines contains a third-order L-C type pole at dc as compared with a single L-C pole for the interdigital filter with short-circuited terminating lines. This would indicate a preference for the open-circuited terminating line configuration. However, this form has a serious disadvantage in that the Z_{0k} are often large for both wide and moderate band-

width filters. This requires the use of conductors with a small cross-sectional area, and for the case of wide-band filters it requires extremely small line spacings.

In addition to the poor mechanical properties of this physical configuration, the assumption that no significant coupling exists between nonadjacent conductors is not always accurate. The interdigital filter with short-circuited terminating lines often has lower values of Z_{0k} than the comparable interdigital form with open-circuited terminating lines. Series S-plane capacitors sometimes can be incorporated into the end lines, resulting in a prototype identical to the interdigital filter with open-circuited terminating lines in a mechanically superior physical form. For the design of wide-band filters, both network types should be investigated. In many instances, a combination coaxial-interdigital realization such as that of the design example is extremely practical.

2) *Filters of narrow bandwidths:* Both basic interdigital filter forms provide similar performance for narrow bandwidths in that the filtering properties of L-C elements are, for all practical purposes, identical to those of unit elements [9]. In both filter types, however, use of the full number of elements available leads to impractical dimensions. The problem is solved by choosing the end elements arbitrarily to obtain a convenient impedance level throughout the filter. Referring to Tables I and II, this requires arbitrarily choosing C for the configuration with open-circuited terminating lines and Z_1 for the network with short-circuited terminating lines.[12] Thus, for narrow bandwidths, the N-line filter realizes N-2 sections. The values required for C and Z_1, respectively, lead to responses that are very broad so that they do not affect significantly the narrow response produced by the N-2 interior sections.[13] By removing the effects of C and Z_1 to obtain practical dimensions, the interior sections of both basic network configurations become identical. The proper prototype element values for the interior sections of an N-line filter with an optimum attenuation characteristic are obtained as outlined in Reference [9] with $m=1$ and $n=N$-3.

For filters of narrow bandwidth and/or a large number of sections, the element values for the nonredundant prototype become extremely small, especially in the center of the filter. Many transformations of the capacitance matrix are then required to obtain a practical network realization. This difficulty can be overcome by the initial synthesis of a redundant network that has convenient values, using a procedure similar to that given in Reference [5], pp. 109–110. Variations in network form can be obtained by transforming the corresponding redundant capacitance matrix. Tables of element values are presently being computed for both redundant and nonredundant prototypes. (See footnote 9.)

A subject of importance in the practical design aspects of interdigital filters is the relationship of the theory and design methods presented here to Matthaei's approximate design procedure [1], [11]. The advantages and disadvantages of the two methods will be discussed with respect to accuracy, simplicity, flexibility, and theoretical considerations.

1) *Accuracy:* The theory presented in this paper is exact[14] and should be capable of providing a high degree of accuracy. Negligible bandwidth shrinkage was obtained in the experimental model, as compared with a nominal shrinkage of about 7 percent produced by the approximate method. As in the design of any practical microwave component, slight adjustments are always required, but it is anticipated that the exact design approach will provide better results, especially for wide-bandwidth designs. A further advantage of the exact approach is that it allows the use of optimum filter theory [9] and thus makes possible the realization of an optimum transmission characteristic.
2) *Simplicity:* The approximate design method makes use of readily available tables, while the exact method requires the use of special transfer functions and a knowledge of modern network synthesis techniques. The use of tables of element values presently being computed makes the exact design procedure very simple, but until these are readily available, the exact design procedure is limited to those people who have access to a digital computer. A further disadvantage of the exact approach is that each new network type, i.e., doubly terminated designs, singly terminated designs, linear phase designs, etc., requires a new set of computer data while the approximate design methods can use existing tables.
3) *Flexibility:* Great flexibility of network configuration is achieved by the use of the capacitance matrix. Fortunately the advantages of this approach apply equally well to both the exact and the approximate design techniques. Once a set of self and mutual capacitors is obtained, by either the exact or the approximate method, the capacitance matrix can be written by inspection and the equivalent networks determined.
4) *Theoretical Considerations:* It is expected that the application of exact synthesis methods and the

[12] Again, these statements apply directly to symmetric filters, with similar results holding for the asymmetric case.

[13] For the filter with short-circuited terminating lines, if Z_1 is chosen equal to the terminating impedance, the transmission power response realized is exactly that of the N-2 interior sections.

[14] It is assumed that no direct coupling exists between nonadjacent lines. See Section II.

capacitance matrix approach to coupled line networks will be useful in the design of many microwave networks other than the interdigital filter. The methods presented provide detailed insight into possible physical realizations and enable the designer to obtain networks with optimum performance.

VII. Conclusions

The synthesis method discussed based on the impedance matrix provides a general procedure for obtaining exact equivalent circuits. Given an N-port impedance matrix, an equivalent circuit can be derived which exactly describes the network behavior between any two ports. While this procedure is of a very general nature, it is often tedious to apply and becomes practically intractable for complicated networks.

For the case of an array of parallel-coupled lines, the use of the capacitance matrix associated with the two-dimensional network geometry allows an exact equivalent circuit to be obtained in a simplified manner. Transformations of the capacitance matrix which preserve the two-port transmission properties are applied to obtain a network in which an equivalent circuit can be obtained by inspection.[15]

The use of the capacitance matrix method allows the realization of an infinite number of equivalent circuits in a wide variety of physical forms. The examples presented demonstrate the simplicity and flexibility of this design approach. Use of the capacitance matrix in conjunction with tables of prototype element values will make possible the simplified design of filter networks in a large number of different physical forms. The techniques presented might also be applied to more complicated coupled line array configurations, including those in which significant coupling exists between nonadjacent conductors.

The specific examples given are directed toward the design of interdigital band-pass filters; however, the general techniques presented can be used to analyze and design a much broader class of microwave networks. Exact equivalent circuits for other configurations such as the meander line and hairpin line can be obtained by applying different boundary conditions to the basic array of parallel conductors. Once the exact S-plane equivalent circuit is determined, the techniques of modern network synthesis can be used to design matching networks, phase shifters, phase equalizers, and many other microwave devices.

The measured performance of a 3:1 bandwidth interdigital filter showed very close agreement with the theoretical performance. Relatively wide line spacings were obtained by using a combination coaxial-interdigital realization.

[15] The transformation of the capacitance matrix presented preserves the two-port performance for interdigital line networks but does not necessarily do so for arrays with other than open-circuited or short-circuited interior port boundary conditions.

Appendix

The invariance of the products of coupling factors insures that transformations of the capacitance matrix preserve the two-port transmission properties of the corresponding interdigital line. The validity of this statement may be demonstrated as follows.

The two-port impedance parameters of a general N-line interdigital array as determined by the method of Section III are functions of the coupling factors and end line impedances as given in (21):

$$\begin{aligned} z_{11} &= Z_{01} f_{11}(K_i^2, S) \qquad i = 1, 2, \cdots, N-1 \\ z_{22} &= Z_{0N} f_{22}(K_i^2, S) \qquad K_i^2 = (K_{k,k+1})(K_{k+1,k}) \\ z_{12} &= Z_{01} \prod_{k=1}^{N-1} K_{k,k+1} f_{12}(K_i^2, S) \\ z_{21} &= Z_{0N} \prod_{k=1}^{N-1} K_{k+1,k} f_{12}(K_i^2, S). \end{aligned} \tag{21}$$

From (21) it follows that maintaining the impedance levels of the outer lines and performing transformations that leave the products of the coupling factors (K_i^2) invariant does not change the open-circuit driving point impedances z_{11} and z_{22}.

For the transfer impedances z_{12} and z_{21}, reciprocity requires that

$$Z_{01} \prod_{k=1}^{N-1} K_{k,k+1} = Z_{0N} \prod_{k=1}^{N-1} K_{k+1,k}. \tag{22}$$

Maintaining Z_{01} and Z_{0N} gives

$$\frac{\prod_{k=1}^{N-1} K_{k,k+1}}{\prod_{k=1}^{N-1} K_{k+1,k}} = \frac{Z_{0N}}{Z_{01}} = \text{constant}. \tag{23}$$

But

$$\left(\prod_{k=1}^{N-1} K_{k,k+1}\right)\left(\prod_{k=1}^{N-1} K_{k+1,k}\right) = \prod_{i=1}^{N-1} K_i^2 = \text{constant}. \tag{24}$$

Equations (23) and (24) require that both products in (22) be constant, resulting in the invariance of the transfer impedances.

Acknowledgment

A definite pattern was found in the relationship between the parameters of the interdigital filter and its cascade equivalent circuit as obtained by the synthesis technique of Section III. This pattern, described by M. C. Horton, encouraged the author to investigate alternate methods of obtaining the equivalent circuit. The author is further indebted to Mr. Horton for a number of suggestions which added to the clarity of this paper.

Appreciation is also expressed to T. E. Gordon for his work in testing the experimental filter, and to N. O. Tiffany for programming the digital computer.

References

[1] G. L. Matthaei, "Interdigital band-pass filters," *IEEE Trans. on Microwave Theory and Techniques*, vol. MTT-10, pp. 479–491, November 1962.

[2] J. T. Bolljahn and G. L. Matthaei, "A study of the phase and filter properties of arrays of parallel conductors between ground planes," *Proc. IRE*, vol. 50, pp. 299–311, March 1962.

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A Rectangular-Waveguide Filter Using Trapped-Mode Resonators

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***Abstract*—A two-resonator, narrow-band waveguide filter with a very wide stop band is described. Each resonator cavity has one side wall which is entirely open except for a bifurcating *E*-plane septum. Energy in most modes tends to radiate freely out of the open end of each resonator to absorbing material; however, energy in the fundamental TE_{101}-mode is trapped in the resonator structures to give high-*Q* resonances such as are typical of conventional solid-wall resonators. Thus, a primary pass band is obtained similar to that of filters using conventional cavity resonators, but the many higher-order pass bands usually found in cavity-resonator filters are largely eliminated because the higher-order-mode cavity resonances are damped out. This type of filter attenuates unwanted signals mainly by reflection. For applications where a low-input VSWR is desired in the pass band, a bifurcated section of guide backed by absorbing material is also used in the input waveguide so as to tend to absorb the input energy at frequencies above that of the pass band.**

General

A MAJOR DISADVANTAGE of narrow-band cavity-resonator filters is that they have many unwanted (or "spurious") pass bands. These spurious responses occur for higher-order longitudinal resonances of the dominant mode, and for undesired higher-order modes of propagation that are often strongly excited in narrow-band filters. In particular, for high-power systems where insertion loss must be minimal and where arcing in waveguide is always a danger, the problem is compounded because design techniques that improve power handling and reduce pass-band insertion loss tend to increase the number of spurious responses. The reason for this is that high-power handling capability and high unloaded *Q* (for low dissipation loss) are obtained by using cavities of large volume, which can then support many spurious resonances unless selective damping is applied. Our purpose here is to describe a new type of filter that uses resonators that damp out undesirable spurious resonances, with little or no effect on the fundamental resonance. A cross section of this new rectangular-waveguide filter is shown in Fig. 1. Filters similar in principle but which utilize the circular TE_{01}-mode are discussed elsewhere [1].

Attenuation in the stop band is due largely to reflection at the coupling apertures, and partly to selective absorption created by the loss mechanism, which is provided by absorbing material behind reduced-width waveguides opening into a wall of each cavity.[1] There

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